

- c) parallel to y-axis d) parallel to x-axis
6. The general solution of the DE $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ is [1]
 a) $\tan^{-1} \frac{y}{x} = \log x + C$ b) $\tan^{-1} \frac{y}{x} = \log y + C$
 c) $\tan^{-1} \frac{x}{y} = \log x + C$ d) none of these
7. The point which does not lie in the half plane $2x + 3y - 12 \leq 0$ is [1]
 a) (2,1) b) (-3, 2)
 c) (1, 2) d) (2, 3)
8. What is the value of λ for which [1]
 $(\lambda \hat{i} + \hat{j} - \hat{k}) \times (3\hat{i} - 2\hat{j} + 4\hat{k}) = (2\hat{i} - 11\hat{j} - 7\hat{k})$
 a) 2 b) 7
 c) 1 d) -2
9. $\int_0^{\frac{\pi}{6}} \frac{\cos 2x}{(\cos x - \sin x)^2} dx$ [1]
 a) $-\log\left(\frac{\sqrt{3}-1}{2}\right)$ b) $\ln 2 - \ln \sqrt{3}$
 c) $\log\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$ d) $-\log\left(\frac{\sqrt{3}+1}{2}\right)$
10. If A and B are symmetric matrices of order n ($A \neq B$), then [1]
 a) A + B is skew symmetric b) A + B is a diagonal matrix
 c) A + B is a zero matrix d) A + B is symmetric
11. Feasible region shaded for a LPP is shown in figure. Maximum of $Z = 2x + 3y$ occurs at the point [1]
-
- a) (0, 0) b) (4, 0)
 c) none of these d) (0, 4)
12. The vectors $\hat{i} - 2x\hat{j} - 3y\hat{k}$ and $\hat{i} + 3x\hat{j} + 2y\hat{k}$ are orthogonal to each other. Then, the locus of the point (x, y) is [1]
 a) parabola b) hyperbola
 c) circle d) ellipse
13. If A is a 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' is equal to [1]
 a) I b) B^{-1}
 c) I + B d) $(B^{-1})'$
14. If $P(A) = \frac{1}{2}$, $P(B) = 0$, then $P(A|B)$ is [1]
 a) 0 b) not defined

- c) $\frac{1}{2}$ d) 1
15. The general solution of the differential equation $(e^x + 1) y dy = (y + 1) e^x dx$ is: [1]
 a) $y + 1 = e^x + 1 + k$ b) $(y + 1) = k (e^x + 1)$
 c) $y = \log \left\{ \frac{e^x + 1}{y + 1} \right\} + k$ d) $y = \log \{k (y + 1) (e^x + 1)\}$
16. The value of p for which $p(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is: [1]
 a) $\frac{1}{\sqrt{3}}$ b) $\sqrt{3}$
 c) 1 d) 0
17. Let $f(x) = |x|$ and $g(x) = |x^3|$, then [1]
 a) $f(x)$ and $g(x)$ both are differentiable at $x = 0$ b) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$
 c) $f(x)$ and $g(x)$ both are continuous at $x = 0$ d) $f(x)$ and $g(x)$ both are not differentiable at $x = 0$
18. If O is the origin, $OP = 3$ with direction ratios proportional to - 1, 2, - 2 then the coordinates of P are [1]
 a) (3, 6, - 9) b) (1, 2, 2)
 c) (- 1, 2, - 2) d) $(\frac{-1}{9}, \frac{2}{9}, \frac{-2}{9})$
19. **Assertion (A):** If two positive numbers are such that sum is 16 and sum of their cubes is minimum, then numbers are 8, 8. [1]
Reason (R): If f be a function defined on an interval I and $c \in I$ and let f be twice differentiable at c, then $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$ and $f(c)$ is local minimum value of f.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** A function $f: Z \rightarrow Z$ defined as $f(x) = x^3$ is injective. [1]
Reason (R): A function $f: A \rightarrow B$ is said to be injective if every element of B has a pre-Image in A.
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.
 c) A is true but R is false. d) A is false but R is true.

Section B

21. $\cot^{-1}(\sqrt{3})$ [2]
 OR
 $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$
22. Find the local maxima and local minima, if any. Find also the local maximum and the local minimum values, as the case may be: $f(x) = \frac{1}{x^2 + 2}$ [2]
23. Find the intervals in which $f(x) = 2 \log(x - 2) - x^2 + 4x + 1$ is increasing or decreasing. [2]
 OR
 Prove that the function does not have maxima or minima: $h(x) = x^3 + x^2 + x + 1$
24. Evaluate: $\int \frac{1}{4x^2 + 12x + 5} dx$ [2]

25. Find the interval of the function that is strictly increasing or decreasing: $10 - 6x - 2x^2$ [2]

Section C

26. Evaluate: $\int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} dx$ [3]
27. A man is known to speak truth 7 out of 10 times. He threw a pair of dice and reports that a doublet appeared. Find the probability that it was actually a doublet. [3]
28. Evaluate: $\int_0^3 (x + 4) dx$ [3]

OR

Evaluate the integral: $\int \sqrt{\cot \theta} d\theta$

29. The temperature T of a cooling object drops at a rate proportional to the difference T - S, where S is a constant temperature of the surrounding medium. If initially T = 150°C, find the temperature of the cooling object at any time t. [3]

OR

Solve the differential equation: $2xy \frac{dy}{dx} = x^2 + y^2$

30. Solve the Linear Programming Problem graphically: [3]

Maximize $Z = x + y$ Subject to

$$-2x + y \leq 1$$

$$x \leq 2$$

$$x + y \leq 3$$

$$x, y \geq 0$$

OR

Find the minimum value of $3x + 5y$ subject to the constraints

$$-2x + y \leq 4,$$

$$x + y \geq 3,$$

$$x - 2y \leq 2,$$

$$\text{with } x, y \geq 0.$$

31. If $x = a \cos \theta$, $y = b \sin \theta$, show that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$ [3]

Section D

32. Using integration, find the area of the region bounded by the line $x - y + 2 = 0$, the curve $x = \sqrt{y}$ and Y-axis. [5]
33. Let $A = \{1, 2, 3, \dots, 9\}$ and R be the relation in $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation and also obtain the equivalence class $[(2, 5)]$. [5]

OR

Give an example of a map

i. which is one-one but not onto

ii. which is not one-one but onto

iii. which is neither one-one nor onto.

34. An amount of Rs 5000 is put into three investments at the rate of interest of 6%, 7% and 8% per annum respectively. The total annual income is Rs 358. If the combined income from the first two investments is Rs 70 more than the income from the third, find the amount of each investment by matrix method. [5]

35. Find the distance of a point $(2, 4, -1)$ from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$. [5]

OR

Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+1}{-2}$ intersect and find their point of intersection.

Section E

36. **Read the text carefully and answer the questions:** [4]

For an audition of a reality singing competition, interested candidates were asked to apply under one of the two musical genres-folk or classical and under one of the two age categories-below 18 or 18 and above.

The following information is known about the 2000 application received:

- i. 960 of the total applications were the folk genre.
- ii. 192 of the folk applications were for the below 18 category.
- iii. 104 of the classical applications were for the 18 and above category.

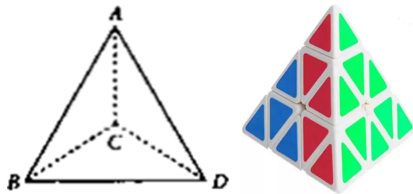
- (i) What is the probability that an application selected at random is for the 18 and above category provided it is under the classical genre? Show your work.
- (ii) An application selected at random is found to be under the below 18 category. Find the probability that it is under the folk genre. Show your work.
- (iii) If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is equal to

OR

If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then find $P(A' \cap B')$.

37. **Read the text carefully and answer the questions:** [4]

A building is to be constructed in the form of a triangular pyramid, ABCD as shown in the figure.



Let its angular points are $A(0, 1, 2)$, $B(3, 0, 1)$, $C(4, 3, 6)$ and $D(2, 3, 2)$ and G be the point of intersection of the medians of $\triangle BCD$.

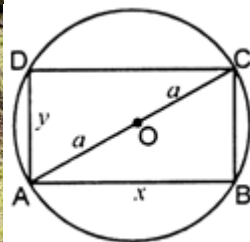
- (i) Find the coordinates of point G
- (ii) Find the length of vector \vec{AG} .
- (iii) Find the area of $\triangle ABC$ (in sq. units).

OR

Find the length of the perpendicular from the vertex D on the opposite face.

38. **Read the text carefully and answer the questions:** [4]

A gardener wants to construct a rectangular bed of garden in a circular patch of land. He takes the maximum perimeter of the rectangular region as possible. (Refer to the images given below for calculations)



- (i) Find the perimeter of rectangle in terms of any one side and radius of circle.
- (ii) Find critical points to maximize the perimeter of rectangle?